Acknowledgments

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Modal Damping Enhancement in Large Space Structures Using AMCD's

70004

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Introduction

LARGE lightweight space structures tend to have extremely low-frequency, lightly damped bending modes. The modal frequencies are usually close together, thus compounding the control problem. The desirability of enhancement of natural damping in large space structures (LSS's) has been discussed in the literature.[1] In this Note, the use of an annular momentum control device (AMCD) is proposed for damping enhancement in LSS's. The basic AMCD concept (shown schematically in Fig. 1) is that of rotating thin rim which is suspended by noncontacting magnetic suspension stations and driven by a noncontacting electromagnetic spin motor. A detailed discussion of the rationale for the AMCD configuration and some of its potential applications are presented in Ref. 2. The control concept presented in this paper is guaranteed to be stable regardless of the number of modes in the LSS model. Also, knowledge of the LSS model is not required.

Linearized Equations of Motion of AMCD/LSS

It is assumed that a single AMCD rim is suspended on the LSS using $l(\geq 3)$ magnetic actuators with infinite bandwidth fixed to the LSS along a circle of radius r, the rim radius. The AMCD is assumed to be very small in diameter (≈ 2 m) compared to the LSS, which means that the rim modes would be well above the LSS's frequencies of interest. Therefore, for the purpose of this development, the AMCD rim is assumed to be rigid. The location of the actuators (and co-located rim position sensors) on the LSS is assumed to be arbitrary for generality. The two-axis linearized equations of motion of the AMCD are given by

$$I_a\ddot{\alpha}_a + W\dot{\alpha}_a = C_1 f \tag{1}$$

$$m_a\ddot{Z}_a = \sum_{i=1}^{l} F_i \tag{2}$$

where I_a is the transverse-axis rim inertia matrix, $\alpha_a=(\phi_a,\theta_a)^T$ are the rim rotation angles about the X and Y axes (see Fig. 1), C_1 is the appropriate moment-arm matrix of actuator locations, m_a is the rim mass, F_i is the axial force (Z-direction) generated by the ith magnetic actuator, $f=(F_1,F_2,\ldots,F_l)^T$, and

$$W=\begin{bmatrix} 0 & H \\ -H & 0 \end{bmatrix} \tag{3}$$

where H is the rim angular momentum about the Z axis. Z_a is the inertial Z-axis position of the rim center. Only x- and y-axis rotations and Z-axis translation are considered in this Note, which suffices to demonstrate the principles. The flexible motion of the LSS is given by

$$\ddot{q}+D\dot{q}+\Lambda q=-\phi^T f \tag{4}$$

where q is the n_q-dimensional vector of modal amplitudes, D is a symmetric positive semidefinite matrix of inherent damping, and Φ is the $l\times n_q$ matrix of mode shapes.

$$\Lambda=\mathrm{diag}(\omega_1^2,\omega_2^2,\ldots,\omega_{n_q}^2) \tag{5}$$

where diag ( ) denotes a diagonal matrix with arguments as entries, and ω_i is the natural frequency of the ith mode. The rigid-body motion of the LSS is not considered since it will be controlled by the primary controller.

The rim is nominally centered within the magnetic actuators. The axial rim centering errors at the l actuator stations are given by

$$\delta=(\delta_1,\delta_2,\ldots\delta_l)^T=\Phi_q-C_1^T\alpha_a-C_2^TZ_a=-\beta^Tx \tag{6}$$

δ_i being the centering error at actuator i, $\beta=[C_1^T,C_2^T,-\Phi]$, where

$$C_2=[1,1,\ldots 1]_{1\times l} \qquad x=(\alpha_a^T,Z_a,q^T)^T \tag{7}$$

The equations of motion are compactly written as

$$A\ddot{x}+B\dot{x}+Cx=\beta f \tag{8}$$

where

$$A=\mathrm{diag}(I_{a_{2\times 2}},m_a,I_{n_q\times n_q}) \qquad B=\mathrm{diag}(W_{2\times 2},0,D_{n_q\times n_q})$$

$$C=\mathrm{diag}(0_{3\times 3},\Lambda_{n_q\times n_q}) \qquad \beta=[C_1^T,C_2^T,-\Phi]^T$$

and where $I_{k\times k}$ denotes the $k\times k$ unit matrix.

The objective of the control system is to produce the magnetic actuator force vector f in such a way that the coupled system given by Eq. (8) is stable. Consider a control law of the type

$$f=K_p\delta+K_r\dot{\delta} \tag{9}$$

where K_p and K_r are $l\times l$ symmetric matrices.

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The closed-loop system is then given by

$$A\ddot{x}+\bar{B}\dot{x}+\bar{C}x=0 \tag{10}$$

where

$$\bar{B}=B+\beta K_r\beta^T \tag{11}$$

$$\bar{C}=C+\beta K_p\beta^T \tag{12}$$

Lemma: The matrix C of Eq. (12) is positive definite if K_p is positive definite.

Proof: From Eq. (12), denoting $L=[C_1^T,C_2^T]^T$

$$\bar{C}=\begin{bmatrix} LK_pL^T & -LK_p\Phi \\ -\Phi^TK_pL^T & \Phi^TK_p\Phi+\Lambda \end{bmatrix} \tag{13}$$

It can be verified that $\bar{C}=E^T\hat{C}E$, where E represents an elementary transformation matrix, and $\hat{C}$ is obtained from $\bar{C}$ by interchanging the (1,1) and (2,2) submatrices, and the (1,2) and (2,1) submatrices. $\bar{C}$ is positive definite if and only if $\hat{C}$ is positive definite. Sylvester's criterion can be used to investigate the positive definiteness of $\hat{C}$. If $K_p>0$, then $\Phi^TK_p\Phi+\Lambda>0$ since $\Lambda>0$. Also, if $K_p>0$, then $LK_pL^T>0$ (since L is of full rank), and all the successive principal minors of LK_pL^T are positive. Let L_k denote the matrix formed by the first k rows of L. Then, the (n_q+k)th principal minor of $\hat{C}$ is given by

$$M_{n_q+k}=\begin{vmatrix} \Phi^TK_p\Phi+\Lambda & -\Phi^TK_pL_k^T \\ -L_kK_p\Phi & L_kK_pL_k^T \end{vmatrix}$$

After expansion and rearrangement,[3] M_{n_q+k} can be written as

$$M_{n_q+k}=(|\Lambda+\Phi^TK^T\{I-KL_k^T(L_kK^TKL_k^T)^{-1}\times L_kK^T\}K\Phi|)(|L_kK_pL_k^T|) \tag{14}$$

where $|\ |$ denotes a determinant, and K_p has been expressed as $K_p=K^TK$, where K is a $l\times l$ nonsingular matrix.

It has been proved in the Appendix that eigenvalues of the matrix $KL_k^T(L_kK^TKL_k^T)^{-1}L_kK^T$ can only be either zero or 1. Therefore, the first determinant in Eq. (14) is positive (since $\Lambda>0$). As stated previously, $|L_kK_pL_k^T|>0$. Therefore, $M_{n_q+k}>0$ for $k=1,2$, and 3, and $\hat{C}$ and $\bar{C}$ are positive definite.

The following theorem states that an AMCD cannot destabilize the system under certain conditions on the gain matrices. Furthermore, the system is asymptotically stable under certain conditions.

Theorem: The closed-loop system of Eq. (10) is stable in the sense of Lyapunov if $K_p>0$ and $K_r\geq 0$. Furthermore, if the number of actuators is at least (n_q+3), and if the rank of β is n_q+3, the system is asymptotically stable provided that $K_r>0$.

Proof: Consider a Lyapunov function

$$V(x,\dot{x})=x^T\bar{C}x+\dot{x}^TA\dot{x} \tag{15}$$

Since $A=A^T>0$ and $\bar{C}>0$ (from the Lemma), V is positive definite. It can be verified using Eq. (10) that

$$\dot{V}=2\dot{x}^T\bar{B}\dot{x}=-\dot{x}^T(\bar{B}+\bar{B}^T)\dot{x} \tag{16}$$

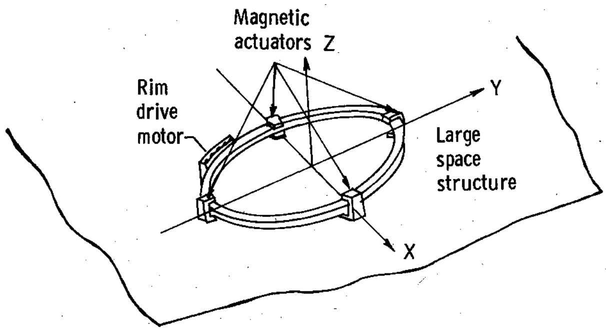

Fig. 1 AMCD/LSS configuration.

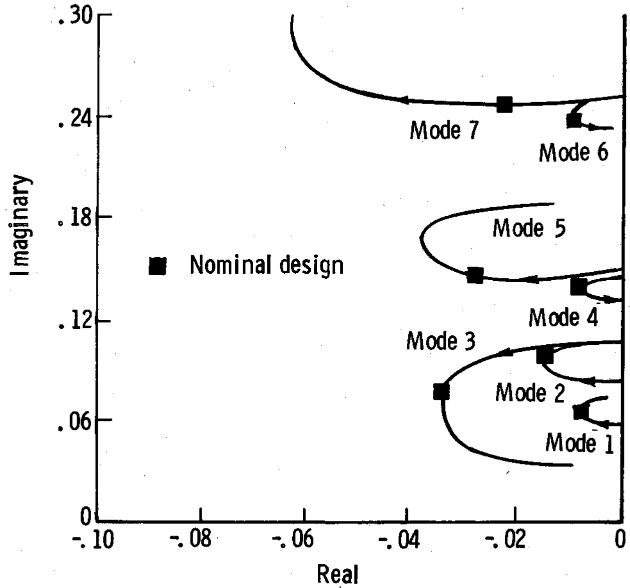

Fig. 2 Variation of closed-loop eigenvalues with K_r.

The last step is a result of the fact that a scalar is equal to its own transpose, i.e., $\dot{x}^T\bar{B}\dot{x}=\dot{x}^T\bar{B}^T\dot{x}$.

From Eq. (11), since W is skew-symmetric,

$$\bar{B}+\bar{B}^T=B+B^T+2\beta K_r\beta^T=2[\mathrm{diag}(0_{3\times 3},D)+\beta K_r\beta^T]$$

Therefore, if $K_r\geq 0$, then $\dot{V}\leq 0$ and the system is stable in the sense of Lyapunov. If $K_r>0$, $l\geq n_q+3$, and the rank of β is n_q+3, then $\beta K_r\beta^T>0$. Therefore, $\dot{V}\leq 0$. Since $\dot{V}\neq 0$ along any trajectories, the system is asymptotically stable.

Remark 1: In practice, it is not possible to use n_q+3 actuators. Even though the sufficient condition for mere stability is satisfied in practice, the system can still be asymptotically stable, as will be demonstrated in a following example.

Remark 2: The AMCD rim angular momentum appears only in the skew-symmetric matrix B, and therefore, does not affect the theorem. That is, the sufficient condition for stability can be satisfied even at zero spin speed (momentum). However, the damping enhancement is negligible at zero momentum. In fact, momentum is very important for getting good modal damping, as discussed in the numerical example given in the following section.

Damping Enhancement in a Completely Free Plate

In order to investigate the concept presented, a 44-mode finite-element model of a completely free, undamped, 30.48-m × 30.48-m × 2.54-mm (100-ft × 100-ft × 0.1-in.) aluminum plate was selected. An AMCD rim of diameter 1.79 m (5.88 ft), mass 34 kg (weight 75 lb), and spin speed of 5000 rpm was assumed to be suspended using four equally spaced magnetic actuator stations. The nominal rim center was arbitrarily located on the plate.

Choosing K_p and K_r to be diagonal matrices with equal entries (k_p and k_r, respectively), root loci were generated by varying k_p and k_r. Figure 2 shows a typical root locus (first seven modes) for $k_p = 146$ N/m and k_r varied from 0 to 19,919 N-s/m. All closed-loop eigenvalues were inside the left half-plane for $k_p > 0$ and $k_r > 0$. Thus, although the sufficient condition for mere stability was satisfied, the system is asymptotically stable. The closed-loop damping ratios (for nominal design) for modes 1, 2, 3, 5, and 7 were 0.13, 0.13, 0.44, 0.2, and 0.1, respectively. Damping on modes 4 and 6 does not improve much because the actuator location is not favorable for these modes. In practice, actuator locations must be chosen so as to get the maximum effect on the most important modes. Although only the first seven modes are shown, all other modes also behave in the same manner.

In order to demonstrate the importance of angular momentum H, gains k_p and k_r were fixed at $k_p = 146$ N/m and $k_r = 5615$ N-s/m, and H was varied from 0 to 4 times its nominal value. At $H = 0$, there is very little damping (only because of the small mass of the rim). The damping improved as H increased, and exhibited a turnaround for some of the modes.

Conclusions

This paper proposes the use of an annular momentum control device (AMCD) for enhancing the modal damping of large space structures (LSS's) during fine pointing missions. It has been proved that an AMCD cannot destabilize the LSS. The control concept presented requires no knowledge of the LSS model, and is stable regardless of the number of modes in the model. Numerical results obtained for a large, thin, completely free undamped aluminum plate indicate that an asymptotically stable closed-loop system can be obtained with satisfactory modal damping ratios. This damping enhancement system, when used in conjunction with a primary controller that controls the LSS rigid-body modes and selected structural modes, has significant potential.

Appendix

Lemma: Let P be an $n \times m$ matrix ($m \leq n$) of rank m. If λ is an eigenvalue of $P(P^TP)^{-1}P^T$, then $\lambda = 0$ or $\lambda = 1$.

Proof: Consider the eigenvalue equation

$$P(P^TP)^{-1}P^Tx = \lambda x \qquad (A1)$$

Premultiplying by P^T,

$$P^Tx = \lambda P^Tx \qquad (A2)$$

Therefore, if $P^Tx \neq 0$, then $\lambda = 1$. If $P^Tx = 0$ and $x \neq 0$, then from Eq. (A1), $\lambda = 0$.

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Technical Comments

G 80 - ~~081~~ 085

Comment on "Method of Optimizing the Update Intervals in Hybrid Navigation Systems"

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LEONDES, Phillis, and Chin[1] have calculated a "lower bound" on the measurement update interval for a hybrid navigation Kalman filter. The proposed lower bound is the smallest update interval for which the innovation sequence $\delta\tilde{z}(k) = \delta z(k) - H(k)\ \delta\hat{x}(k)$ is not significantly time-correlated. The authors state that, if the update interval is smaller than this lower bound, the process model will be invalid because the assumption of white measurement noise will be unjustified. This would be correct if the measurement noise were assumed to be white. However, this whiteness assumption is not necessary (nor is it physical, as the authors point out), and in fact Kalman filters may be synthesized when state noise and measurement noise are assumed to be colored.[2] For a filter so designed, time-correlation of the sequence $\delta\tilde{z}(k)$ is expected and is not indicative of invalidity of the model. If, on the other hand, a Kalman filter has been designed based on an assumption of white measurement noise, improved performance might be obtained by redesigning the filter to model the colored noise and then descreasing the update interval to the smallest computationally feasible interval.

The authors also err in stating that "Sampling at time points spaced by Δt is analogous to passing the continuous noise through a bandpass filter of bandwidth $B = 2\pi/\Delta t$." If the sampling is ideal (i.e., impulsive), then there is no attenuation of high-frequency components, and aliasing will occur. More realistically, if the sampling pulse width is small but nonzero, the sampling process will pass (and, in fact, generate) components at frequencies much higher than the sampling frequency.

no ref (TN) G-79-002

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